# HINTS AND ANSWERS

## CHAPTER 1

1.2 Use the definitions.

1.4 To get you started, take the indifference relation. Consider any three points $\mathbf{x}^i \in X, i = 1, 2, 3$, where $\mathbf{x}^1 \sim \mathbf{x}^2$ and $\mathbf{x}^2 \sim \mathbf{x}^3$. We want to show that $\mathbf{x}^1 \sim \mathbf{x}^2$ and $\mathbf{x}^2 \sim \mathbf{x}^3 \Rightarrow \mathbf{x}^1 \sim \mathbf{x}^3$. By definition of $\sim, \mathbf{x}^1 \sim \mathbf{x}^2 \Rightarrow \mathbf{x}^1 \succsim \mathbf{x}^2$ and $\mathbf{x}^2 \succsim \mathbf{x}^1$. Similarly, $\mathbf{x}^2 \sim \mathbf{x}^3 \Rightarrow \mathbf{x}^2 \succsim \mathbf{x}^3$ and $\mathbf{x}^3 \succsim \mathbf{x}^2$. By transitivity of $\succsim, \mathbf{x}^1 \succsim \mathbf{x}^2$ and $\mathbf{x}^2 \succsim \mathbf{x}^3 \Rightarrow \mathbf{x}^1 \succsim \mathbf{x}^3$. Keep going.

1.16 For (a), suppose there is some other feasible bundle $\mathbf{x}'$, where $\mathbf{x}' \sim \mathbf{x}^*$. Use the fact that $B$ is convex, together with strict convexity of preferences, to derive a contradiction. For (b), suppose not. Use strict monotonicity to derive a contradiction.

1.22 Use a method similar to that employed in (1.11) to eliminate the Lagrangian multiplier and reduce $(n + 1)$ conditions to only $n$ conditions.

1.23 For part (2), see Axiom 5′: Note that the sets $\succsim(\mathbf{x})$ are precisely the superior sets for the function $u(\mathbf{x})$. Recall Theorem A1.14.

1.27 Sketch out the indifference map.

1.28 For part (a), suppose by way of contradiction that the derivative is negative.

1.29 Set down all first-order conditions. Look at the one for choice of $x_0^*$. Use the constraint, and find a geometric series. Does it converge?

1.32 Feel free to assume that any necessary derivatives exist.

1.33 Roy's identity.

1.41 Theorem A2.6.

1.46 Euler's theorem and any demand function, $x_i(\mathbf{p}, y)$.

1.47 For part (a), start with the definition of $e(\mathbf{p}, 1)$. Multiply the constraint by $u$ and invoke homogeneity. Let $\mathbf{z} \equiv u\mathbf{x}$ and rewrite the objective function as a choice over $\mathbf{z}$.

1.52 Take each inequality separately. Write the one as

$$\frac{\partial x_i(\mathbf{p}_i, y)/\partial y}{x_i(\mathbf{p}, y)} \leq \frac{\bar{\eta}}{y}.$$

Integrate both sides of the inequality from $\bar{y}$ to $y$ and look for logs. Take it from there.

1.54 For part (b),

$$v(\mathbf{p}, y) = A^* y \prod_{i=1}^{n} p_i^{-\alpha_i},$$

where $A^* = A \prod_{i=1}^{n} \alpha_i^{\alpha_i}$.

1.60 Use Slutsky.

1.63 No hints on this.

1.66 For (b), $u^0$ must be $v(\mathbf{p}^0, y^0)$, right? Rewrite the denominator.

1.67 For (a), you need the expenditure function and you need to figure out $u^0$. For (b), $I = (u^0 - 1/8)/(2u^0 - 1)$. For (c), if you could show that the expenditure function must be multiplicatively separable in prices and utility, the rest would be easy.

## CHAPTER 2

2.3 It should be a Cobb-Douglas form.

2.9 Use a diagram.

2.10 To get you started, $\mathbf{x}^2$ is revealed preferred to $\mathbf{x}^1$.

2.12 For (a), use GARP to show that, unless $\phi(\mathbf{x}^j)$ is zero, there is a minimising sequence of distinct numbers $k_1, ..., k_m$ defining $\phi(\mathbf{x}^j)$ such that no $k_1, ..., k_m$ is equal to $j$. Hence, $k_1, ..., k_m, j$ is a feasible sequence for the minimisation problem defining $\phi(\mathbf{x}^k)$. For (b), use (a). For (c), recall that each $\mathbf{p}^k \in \mathbb{R}^n_{++}$. For (e), the minimum of quasiconcave functions is quasiconcave.

2.13 Let $\mathbf{x}^0 = \mathbf{x}(\mathbf{p}^0, 1), \mathbf{x}^1 = \mathbf{x}(\mathbf{p}^1, 1)$, and consider $f(t) \equiv (\mathbf{p}^0 - \mathbf{p}^1) \cdot \mathbf{x}(\mathbf{p}^1 + t(\mathbf{p}^0 - \mathbf{p}^1), \ (\mathbf{p}^1 + t(\mathbf{p}^0 - \mathbf{p}^1)) \cdot \mathbf{x}^0)$ for $t \in [0, 1]$. Show that if $\mathbf{x}^0$ is revealed preferred to $\mathbf{x}^1$ at $(\mathbf{p}^0, 1)$, then $f$ attains a maximum uniquely at 0 on [0, 1].

2.14 In each of the two gambles, some of the outcomes in $A$ will have zero probability.

2.16 Remember that each outcome in $A$ is also a gamble in $\mathcal{G}$, offering that outcome with probability 1.

2.17 Axiom G4.

2.19 Which of the other axioms would be violated by the existence of two unequal indifference probabilities for the same gamble?

2.28 Risk averse.

2.32 Rearrange the definition and see a differential equation. Solve it for $u(w)$.

2.33 If you index his utility function by his initial wealth, then given two distinct wealth levels, how must the two utility functions be related to one another?

2.34 $u(w) = w^{\alpha+1}/(\alpha + 1)$.

2.38 For (a), $x_0 = x_1 = 1$. For (b), the agent faces *two* constraints, and $x_0 = 1, x_1^H = 3/2$ and $x_1^L = 1/2$. For (c), note that future income in the certainty case is equal to the expected value of income in the uncertain case.

## CHAPTER 3

3.16 First find $MRTS_{ij}$ and write it as a function of $r = x_j/x_i$. Take logs and it should be clear.

3.17 For (a), first take logs to get

$$\ln(y) = \frac{1}{\rho} \ln\left(\sum_{i=1}^{n} \alpha_1 x_i^{\rho}\right).$$

Note that $\lim_{\rho\to 0} \ln(y) = 0/0$, so L'Hôpital's rule applies. Apply that rule to find $\lim_{\rho\to 0} \ln(y)$, then convert to an expression for $\lim_{\rho\to 0} y$. Part (b) is tough. If you become exasperated, try consulting Hardy, Littlewood, and Pólya (1934), Theorem 4.

3.20 Just work with the definitions and the properties of the production function.

3.23 For the second part, let $\mathbf{z}^2 = \Delta\mathbf{z}^1 \geq \mathbf{0}$.

3.32 $c(y) \equiv atc(y)y$.

3.43 Equations (3.3) and (3.4).

3.45 Work from the first-order conditions.

3.50 Define

$$\pi_v(p, \mathbf{w}, \bar{\mathbf{x}}) \equiv \max_{y,\mathbf{x}} py - \mathbf{w}\cdot\mathbf{x} \qquad \text{s.t.} \qquad f(\mathbf{x}, \bar{\mathbf{x}}) \geq y,$$

sometimes called the **variable profit function**, and note that $\pi_v(p, \mathbf{w}, \bar{\mathbf{x}}) = \pi(p, \mathbf{w}, \bar{\mathbf{w}}, \bar{\mathbf{x}}) + \bar{\mathbf{w}}\cdot\bar{\mathbf{x}}$. Note that $\pi_v$ possesses *every* property listed in Theorem 3.7, and that the partial derivatives of $\pi_v$ and $\pi(p, \mathbf{w}, \bar{\mathbf{w}}, \bar{\mathbf{x}})$ with respect to $p$ and $\mathbf{w}$ are equal to each other.

3.55 $K^* = 5\sqrt{w_f/w_k}$.

## CHAPTER 4

4.1 Exercise 1.65.

4.2 Try to construct a counterexample.

4.9 In part (b), $q^1 = 215/6$, $q^2 = 110/6$, and $p = 275/6$.

4.13 $p^1 = p^2 = 80/3$.

4.14 Exploit the symmetry here.

4.15 For (c), $J^*$ is the largest integer less than or equal to $1 + 1/\sqrt{2k}$.

4.18 For (a), just let $\eta(y) = \eta$, a constant, where $\eta \neq 1$. For (b), let $\eta = 0$. For (c), start over from the beginning and let $\eta(y) = 1$. For (d), according to Taylor's theorem, $f(t) \approx f(t_0) + f'(t_0)(t - t_0) + (1/2)f''(t_0)(t - t_0)^2$ for arbitrary $t_0$. Rearrange and view the expression for $CV + y^0$ as the function $y^0[t + 1]^{1/(1-\eta)}$. Apply Taylor's theorem and evaluate at $t_0 = 0$.

4.19 In (b), $v(p, y) = \ln(1/p) + y - 1$. For (d), will anything from Exercise 4.18 help?

4.20 Exercise 4.18.

4.25 For (a), $p^1 = p^2 = 4$. For (b), look at the tail.

4.26 Sketch it out carefully on a set of diagrams like Fig. 4.2. For (d), does it really make any difference to *consumers*? For (e), go ahead and assume that everyone is identical. Still, you will have to think about a lot of things.

## CHAPTER 5

5.2 Differentiate the indirect utility function with respect to the price that rises and use Roy's identity.

5.10 Don't use fancy maths. Just think clearly about what it means to be Pareto efficient and what it means to solve the given set of problems.

5.12 Use $x_2$ as numéraire. For (b), remember that neither consumption nor prices can be negative.

5.15 Derive the consumers' demand functions.

5.16 The function $u^2(\mathbf{x})$ is a Leontief form.

5.17 The relative price of $x_1$ will have to be $\alpha/(1 - \alpha)$.

5.18 For (a), $\mathbf{x}^1 = (10/3, 40/3)$.

5.19 Calculate $\mathbf{z}(\mathbf{p})$ and convince yourself if $\mathbf{p}^*$ is a Walrasian equilibrium, then $\mathbf{p}^* \gg \mathbf{0}$. Solve the system of excess demand functions.

5.20 For (b), remember that total consumption of each good must equal the total endowment. Suppose that $\bar{p}$ is a market-clearing relative price of good $x_1$, but that $\bar{p} \neq p^*$. Derive a contradiction.

5.21 Consider the excess demand for good 2 when the price of good one is positive, and consider the excess demand for good one when the price of good one is zero.

5.22 For part (a), show first that if $u(\cdot)$ is strictly increasing and quasiconcave, then for $\alpha > 0$, $v(\mathbf{x}) = u\left(x_1 + \alpha \sum_{i=1}^n x_i, \ldots, x_n + \alpha \sum_{i=1}^n x_i\right)$ is strongly increasing and quasiconcave. Show next that if $u(\cdot)$ is strongly increasing and quasiconcave, then for $\varepsilon \in (0, 1)$, $v(\mathbf{x}) = u(x_1^\varepsilon, \ldots, x_n^\varepsilon)$ is strongly increasing and strictly quasiconcave. Now put the two together. For part (c), equilibrium prices can always be chosen to be non-negative and sum to one and hence contained in a compact set. Hence, any such sequence has a convergent subsequence.

5.23 See Assumption 5.2 for a definition of strong convexity.

5.26 $(p_y/p_h)^* = 4\sqrt{2}$ and he works an 8-hour day.

5.27 To show proportionality of the gradients, suppose they are not. Let $\mathbf{z} = (\nabla u^i(\bar{\mathbf{x}}^i)/\|\nabla u^i(\bar{\mathbf{x}}^i)\|) - (\nabla u^j(\bar{\mathbf{x}}^j)/\|\nabla u^j(\bar{\mathbf{x}}^j)\|)$, and show that $u^i(\bar{\mathbf{x}}^i + t\mathbf{z})$ and $u^j(\bar{\mathbf{x}}^j - t\mathbf{z})$ are both strictly increasing in $t$ at $t = 0$. You may use the Cauchy-Schwartz inequality here, which says that for any two vectors, $\mathbf{v}$ and $\mathbf{w}$, $\mathbf{v} \cdot \mathbf{w} \leq \|\mathbf{v}\|\|\mathbf{w}\|$, with equality if and only if the two vectors are proportional.

5.38 Look carefully at the proof in the text. Construct the coalition of worst off members of *every* type. Give each coalition member the 'average' allocation for his type.

5.39 For (b), translate into terms of these utility functions and these endowments what it means to be (1) 'in the box', (2) 'inside the lens', and (3) 'on the contract curve'. For (d), consider the coalition $S = \{11, 12, 21\}$ and find a feasible assignment of goods to consumers that the consumers in $S$ prefer.

5.40 Redistribute endowments equally. This will be envy-free. Invoke Theorem 5.5 and consider the resulting WEA, $\mathbf{x}^*$. Invoke Theorem 5.7. Now prove that $\mathbf{x}^*$ is also envy-free.

5.41 For (b), see the preceding exercise.

5.42 Fair allocations are defined in Exercise 5.40.

5.43 For (a), indifference curves must be tangent and all goods allocated. For (b), not in general.

5.46 Exercises 1.65 and 4.1 [Actually, this problem only tells you half the story. It follows from *Antonelli's Theorem* that $\mathbf{z}(\mathbf{p})$ is both independent of the distribution of endowments *and* behaves like a single consumer's excess demand system if and only if preferences are identical and homothetic. See Shafer and Sonnenschein (1982) for a proof.]

# Chapter 6

6.2 Show that *VWP* and *IIA* together imply *WP*.

6.4 Here is the proof mentioned in the stem of the question: Suppose we want $u(x^k) = a_k$ for $k = 1, 2, \ldots, m$, where the $x^k$ are distinct members of $X$. Let $2\varepsilon > 0$ be the minimum Euclidean distance between any distinct pair of the $x^k$. Letting $\|\cdot\|$ denote Euclidean distance, define $u(x) = 0$ if for every $k$ we have $\|x - x^k\| \geq \varepsilon$, and define $u(x) = \left(1 - \frac{\|x-x^k\|}{\varepsilon}\right) a_k$ if $\|x - x^k\| < \varepsilon$ for some $k$ (by the triangle inequality there can be at most one such $k$). Prove that $u(\cdot)$ is continuous and that $u(x^k) = a_k$ for every $k$.

6.5 For part (b), what assumptions did we make about $X$? What additional assumptions did we make about social preferences? Do our additional assumptions about individual preferences play a role?

6.8 In part (b), notice that for $\varepsilon > 0$ small enough, $u^j - \varepsilon < u^j < \alpha + \varepsilon < u^i$. Now apply *HE*.

6.9 For part (e), consider changing individual 2's profile in (a) so that it becomes identical to 3's profile. What happens to the social preference between $x$ and $z$?

6.11 For (a), if $\mathbf{x}^* \gg \mathbf{0}$ is a WEA, there must exist $n$ prices $(p_i^*, \ldots, p_n^*)$ such that every $(\mathbf{x}^i)^*$ maximises agent $i$'s utility over their budget set. Look at these first-order conditions and remember that the Lagrangian multiplier for agent $i$ will be equal to the marginal utility of income for agent $i$ at the WEA, $\partial v^i(\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i)/\partial y$. Next, note that $W$ must be strictly concave. Thus, if we have some set of weights $\alpha^i$ for $i \in \mathcal{I}$ and an $n$-vector of numbers $\theta = (\theta_1, \ldots, \theta_n)$ such that $\alpha^i \nabla u^i((\mathbf{x}^i)^*) = \theta$ and $\mathbf{x}^*$ satisfies the constraints, then $\mathbf{x}^*$ maximises $W$ subject to the constraints. What if we choose the

$\alpha^i$ to be equal to the reciprocal of the marginal utility of income for agent $i$ at the WEA? What could we use for the vector $\theta$? Pull the pieces together.

6.12 For (b), consider this three-person, three-alternative case due to Sen (1970a). First, let $xP^1yP^1z$, $zP^2xP^2y$, and $zP^3xP^3y$. Determine the ranking of $x$ versus $z$ under the Borda rule. Next, let the preferences of 2 and 3 remain unchanged, but suppose those of 1 become $x\bar{P}^1z\bar{P}^1y$. Now consider the same comparison between $x$ and $z$ and make your argument.

6.13 Why can't $(x, y)$ and $(z, w)$ be the same pair? If $x = z$, invoke $U$ and suppose that $xP^ky$, $wP^jx$, and $yP^iw$ for all $i$. Use $L^*$ and $WP$ to show that transitivity is violated. If $x$, $y$, $z$, and $w$ are all distinct, let $xP^ky$, $zP^jw$, and suppose that $wP^ix$ and $yP^iz$ for all $i$. Take it from here.

6.15 For (b) and (c), see Exercise A2.10 for the necessary definition. For (e),

$$E(w, \mathbf{y}) = \left( \sum_{i=1}^{N} \frac{1}{N} \left( \frac{y^i}{\mu} \right)^{\rho} \right)^{1/\rho}.$$

6.16 No, no, no, yes.

6.17 For parts (a) and (b), no and yes.

6.19 Argue by contradiction and change preferences monotonically so that all preferences are strict and $x$ is at the top of $n$'s ranking.

## CHAPTER 7

7.3 For part (b), show first that if a strategy is strictly dominated in some round, then it remains strictly dominated by some remaining strategy in every subsequent round.

7.5 For (c), when is 99 a best response? To find $W_i^1$, follow these steps. Step 1: Show that if 14 results in a tie, then 15, 16, ..., 100 either lose or tie. Step 2: Show that 14 wins if all other players choose a number strictly above 14. Step 3: Show that 14 loses only if one-third the average of the numbers is strictly less than 14. Conclude from steps 2 and 3 that if 14 loses, so do 15, 16, ..., 100.

7.7 For (a), use the Nash equilibrium existence theorem.

7.8 Employ a fixed-point function similar to the one used to prove the existence of a Nash equilibrium to prove the existence of a strategy $m^* \in M_1$ for player 1, which maximises $u_1(m, m^*, \ldots, m^*)$ over $m \in M_1$. Then invoke symmetry.

7.9 See Exercise 7.7, part (c), for the definition of a game's value. See Exercise 7.8 for the definition of a symmetric game.

7.21 Would a player ever choose a strictly dominated strategy?

7.22 Do not rule out weakly dominated strategies. Are all of these equilibria in pure strategies? Verify that the high-cost type of a firm 2 earns zero profits in every Bayesian-Nash equilibrium.

7.32 Allow information sets to 'cross' one another.

7.42 Can 3's beliefs about 2 be affected by 1's choice?

# CHAPTER 8

8.1 Recall that wealth in one state is a different commodity from wealth in another state and that the state in which consumer $i$ alone has an accident is different than that in which only consumer $j \neq i$ does. Verify the hypotheses of the First Welfare Theorem and conclude that the competitive outcome of Section 8.1 is efficient in the sense described there.

8.5 For part (c), suppose there are at least three fixed points. Let $x^*$ be a fixed point between two others and think about what would be implied if $f'(x^*) \geq 1$ and what would be implied if $f'(x^*) \leq 1$.

8.7 For (a), suppose not. Could demand for used cars equal supply?

8.12 For part (c), it is not a pooling contract.

8.13 First show that $\sum_{l=k}^{L}(\pi_l(0) - \pi_l(1)) > 0$ for all $k > 0$ by writing the sum instead as $\sum_{l=k}^{L}\left(\frac{\pi_l(0)}{\pi_l(1)} - 1\right)\pi_l(1)$ and arguing by contradiction. Finally, apply the following identity: $\sum_{l=0}^{L} a_l b_l \equiv \sum_{k=0}^{L}\left(\sum_{l=k}^{L} a_l\right)(b_k - b_{k-1})$ for every pair of real sequences $\{a_l\}_{l=0}^{L}, \{b_l\}_{l=0}^{L}$, and where $b_{-1} \equiv 0$.

8.16 For part (a), use the fact that the owner is risk neutral and the worker is risk-averse.

8.17 (a) Because the manager observes only output and not effort, the wage can depend only on output. Let $w(y)$ denote the worker's wage when output is $y$. The manager's problem is therefore as follows:

$$\max_{e, w(y_1), \ldots, w(y_m)} \sum_{i=1}^{m} p(y_i|e)(y_i - w(y_i)),$$

where $e \in \{e_1, \ldots, e_n\}$ and each $w(y_i) \in \mathbb{R}$, subject to

$$\sum_{i=1}^{m} p(y_i|e)u(w(y_i), e) \geq \sum_{i=1}^{m} p(y_i|e_j)u(w(y_i), e_j), \text{ for all } j = 1, \ldots, n$$

and

$$\sum_{i=1}^{m} p(y_i|e)u(w(y_i), e) \geq u(0, e_1).$$

(b) Let $e^* > e_1$ denote the effort level chosen by the worker in the optimal solution. Suppose, by way of contradiction, that the wage contract is non-increasing, i.e., that $w(y_i) \geq w(y_{i+1})$ for all $i$. Then, by the monotone likelihood ratio property, and Exercise 8.13,

$$\sum_{i=1}^{m} p(y_i|e^*)u(w(y_i), e^*) \leq \sum_{i=1}^{m} p(y_i|e_1)u(w(y_i), e^*),$$

because the function $u(w(y), e^*)$ is non-increasing in $y$. However, because $u(w, e)$ is strictly decreasing in $e$, we have

$$\sum_{i=1}^{m} p(y_i|e_1)u(w(y_i), e^*) < \sum_{i=1}^{m} p(y_i|e_1)u(w(y_i), e_1).$$

Putting the two inequalities together yields,

$$\sum_{i=1}^{m} p(y_i|e^*)u(w(y_i), e^*) < \sum_{i=1}^{m} p(y_i|e_1)u(w(y_i), e_1),$$

in violation of the incentive compatibility constraint. This contradiction proves the result.

## CHAPTER 9

9.3 Note that (9.3) holds for all $v$, including $v = r$.

9.7 What are the first- and second-order conditions for bidder $i$ implied by incentive compatibility? Because the first-order condition must hold for all $v_i$, it may be differentiated. Use the derivative to substitute into the second-order condition.

9.13 Did any of our results depend on the values being in [0, 1]?

9.15 (b) What is the induced direct selling mechanism?

9.17 (b) Use Theorem 9.5, and don't forget about individual rationality.

9.19 You will need to use our assumption that each $v_i - (1 - F_i(v_i))/f_i(v_i)$ is strictly increasing.

# MATHEMATICAL APPENDIX
## CHAPTER A1

A1.2 Just use the definitions of subsets, unions, and intersections.

A1.3 To get you started, consider the first one. Pick any $x \in (S \cap T)^c$. If $x \in (S \cap T)^c$, then $x \notin S \cap T$. If $x \notin S \cap T$, then $x \notin S$ or $x \notin T$. (Remember, this is the inclusive 'or'.) If $x \notin S$, then $x \in S^c$. If $x \notin T$, then $x \in T^c$. Because $x \in S^c$ or $x \in T^c$, $x \in S^c \cup T^c$. Because $x$ was chosen arbitrarily, what we have established holds for all $x \in (S \cap T)^c$. Thus, $x \in (S \cap T)^c \Rightarrow x \in S^c \cup T^c$, and we have shown that $(S \cap T)^c \subset S^c \cup T^c$. To complete the proof of the first law, you must now show that $S^c \cup T^c \subset (S \cap T)^c$.

A1.13 To get you started, let $x \in f^{-1}(B^c)$. By definition of the inverse image, $x \in D$ and $f(x) \in B^c$. By definition of the complement of $B$ in $R$, $x \in D$ and $f(x) \notin B$. Again, by the definition of the inverse image, $x \in D$ and $x \notin f^{-1}(B)$. By the definition of the complement of $f^{-1}(B)$ in $D$, $x \in D$ and $x \in (f^{-1}(B))^c$, so $f^{-1}(B^c) \subset (f^{-1}(B))^c$. Complete the proof.

A1.18 Let $\Omega^i = \{\mathbf{x} \mid \mathbf{a}^i \cdot \mathbf{x} + b^i \geq 0\}$. Use part (b) of Exercise A1.17.

A1.21 First, model your proof after the one for part 3. Then consider $\cap_{i=1}^{\infty} A_i$, where $A_i = (-1/i, 1/i)$.

A1.22 Draw a picture first.

A1.24 Look at the complement of each set.

A1.25 Use Theorem A1.2 to characterise the complement of $S$ in $\mathbb{R}$.

A1.26 For the first part, sketch something similar to Fig. A1.12 and use what you learned in Exercise A1.24. The second part is easy.

A1.27 To get you started, note that the complement of $S$ is open, then apply Theorem A1.3. Open balls in $\mathbb{R}$ are open intervals. Use what you learned in Exercise A1.26.

A1.31 Centre a ball at the origin.

A1.32 For part (c), you must show it is bounded *and* closed. For the former, centre a ball at the origin. For the latter, define the sets $F_0 \equiv \{\mathbf{x} \in \mathbb{R}^n | \sum_{i=1}^n x_i = 1\}$, $F_i \equiv \{\mathbf{x} \in \mathbb{R}^n | x_i \geq 0\}$, for $i = 1, \cdots, n$. Convince yourself that the complement of each set is open. Note that $S^{n-1} = \cap_{i=0}^n F_i$. Put it together.

A1.38 Look closely at $S$.

A1.39 Check the image of $f(x) = \cos(x) - 1/2$.

A1.40 Choose a value for $y$, some values for $x_1$, and solve for the values of $x_2$. Plot $x_1$ and $x_2$.

A1.46 In (b), it may help to remember that $\mathbf{x}^1$ and $\mathbf{x}^2$ can be labelled so that $f(\mathbf{x}^1) \geq f(\mathbf{x}^2)$, and that $tf(\mathbf{x}^1) + (1-t)f(\mathbf{x}^2) = f(\mathbf{x}^2) + t(f(\mathbf{x}^1) - f(\mathbf{x}^2))$.

A1.49 Yes, yes, no, yes. Look for convex sets. For (e), things will be a bit different if you assume $f(x)$ is continuous and if you do not.

## CHAPTER A2

A2.1 For (g), $f'(x) = -\exp(x^2) < 0$.

A2.2 For (a), $f_1 = 2 - 2x_1$ and $f_2 = -2x_2$. For (e), $f_1 = 3x_1^2 - 6x_2$ and $f_2 = -6x_1 + 3x_2^2$.

A2.3 Chain rule.

A2.5 For (a),

$$\mathbf{H}(\mathbf{x}) = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}.$$

A2.11 Use the definition of an increasing function and the definitions of local optima.

A2.19 Strict quasiconcavity implies quasiconcavity.

A2.24 For (a), $\mathbf{x}^* = (1, 0)$ is a maximum. For (b), $\mathbf{x}^* = (0, 1)$ is a minimum.

A2.25 (a) $(1, 1)$ and $(-1, -1)$; $f(1, 1) = f(-1, -1) = 2$; (b) $(-\sqrt{1/2}, \sqrt{1/2})$ and $(\sqrt{1/2}, -\sqrt{1/2})$; (c) $(\sqrt{a^2/3}, \sqrt{2b^2/3})$ and $(\sqrt{a^2/3}, -\sqrt{2b^2/3})$; (d) $((1/2)^{1/4}, (1/2)^{1/4})$; (e) $(1/6, 2/6, 3/6)$.

A2.37 Use the fact that sequences in $A$ are bounded and therefore have convergent subsequences.